Additional Solutions – Chapter 4

Section 4.3

1. Vertex $(3,2)$

$$0 = 4(x-3)^2 + 2$$
$$-2 = 4(x-3)^2$$
$$-\tfrac{1}{2} = (x-3)^2$$

Find zeros
$$\sqrt{-\tfrac{1}{2}} = \sqrt{(x-3)^2}$$
$$\pm\sqrt{-\tfrac{1}{2}} = x-3$$
$$x = 3 \pm \sqrt{-\tfrac{1}{2}}$$
$$x = 3 \pm \sqrt{\tfrac{1}{2}}\ i$$

There aren't any real zeros since the solution is a complex number

5. Vertex $(0.5,-2)$

$$0 = 8(x-0.5)^2 - 2$$
$$2 = 8(x-0.5)^2$$

Find zeros
$$\tfrac{1}{4} = (x-0.5)^2$$
$$\sqrt{\tfrac{1}{4}} = \sqrt{(x-0.5)^2}$$
$$\pm 0.5 = x-0.5$$

Break into two equations
$$0.5 = x-0.5 \qquad -0.5 = x-0.5$$
$$1 = x \qquad 0 = x$$

There zeros are $x = 0$ and $x = 1$.

9. Use $x = \dfrac{-b}{2a}$ to find the vertex

$$x = \frac{0}{2(9)} = 0 \qquad f(0) = 9(0)^2 - 4 = -4$$

The vertex is $(0,-4)$

16. The standard form gives the vertical stretch/compression and reflection over the x-axis as well as the vertical intercept. Its disadvantages are that work must be done to find the vertex and the zeros. The vertex form gives the vertex and the vertical stretch/compression and reflection over the x-axis, so additional work is needed to find the zeros and the vertical intercept. While the factored form lists the zeros and vertical stretch/compression and reflection over the x-axis, more work is required to find the vertex and the vertical intercept.

20. The zeros are equal horizontal distance to the left and right from the line of symmetry (x-coordinate of the vertex). Find the difference in the horizontal distance between the known zero and the vertex. Use that distance to determine the other zero.

21. Area of Mat = Mat area – Cut-out area

$A = M - C$

Mat area is $M = 18^2 = 324$

Let $x =$ length of picture, but mat will overlap the picture by 1" on each side

Therefore, Cut-out area is $C = (x-2)^2$

$A(x) = M - C$

$A(x) = 324 - (x-2)^2$

For a 12" x 12" picture	$A(12) = 324 - (12-2)^2$ $= 224$	The mat frame area is 224 sq. inches.
For a 15" x 15" picture	$A(15) = 324 - (15-2)^2$ $= 155$	The mat frame area is 155 sq. inches

22. Area of Mat = Mat area – Cut-out area

$A = M - C$

Mat area is $M = 16^2 = 256$

Let $x =$ length of picture, but mat will overlap the picture by 1" on each side

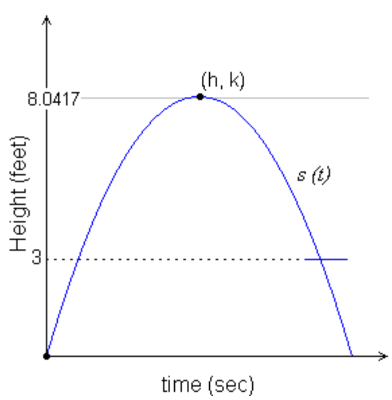
Therefore, Cut-out area is $C = (x-2)^2$

$A(x) = M - C$

$A(x) = 256 - (x-2)^2$

For a 10" x 10" picture	$A(10) = 256 - (10-2)^2$ $= 192$	The mat frame area is 192 sq. inches.
For a 14" x 14" picture	$A(14) = 256 - (14-2)^2$ $= 112$	The mat frame area is 112 sq. inches

23. Height above ground function $s(t) = -16(t-h)^2 + k$ uses feet for the unit



Jump height 8 ft ½ in = 8. 0417 ft

Vertex is $(t,\ 8.0417)$ Additional point $(0,0)$

$$s(t) = -16(t-h)^2 + k$$
$$0 = -16(0-h)^2 + 8.0417$$
$$-8.0417 = -16h^2$$
$$0.5026 = h^2$$
$$h = 0.7089$$

Therefore, $s(t) = -16(t-0.7089)^2 + 8.0417$

Find the time when he’s at a height of 3 ft

$$3 = -16(t-0.7089)^2 + 8.0417$$

$$-5.0417 = -16(t-0.7089)^2$$

$$0.3151 = (t-0.7089)^2$$

$$\pm 0.5613 = t - 0.7089$$

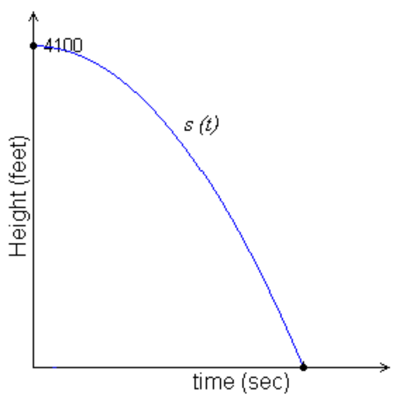
Continue solving for both times

$$0.5613 = t - 0.7089 \qquad -0.5613 = t - 0.7089$$

$$t = 1.2702 \qquad t = 0.1476$$

Sotomayor would have landed in the cushion during the later time, so he was airborne for approximately 1.27 sec.

23. Height of a falling object $s(t) = -16t^2 + s_0$ uses feet for the unit



Initial height is 4100 ft $s_0 = 4100$

$$s(t) = -16t^2 + 4100$$

Find when $s(t) = 0$

$$0 = -16t^2 + 4100$$

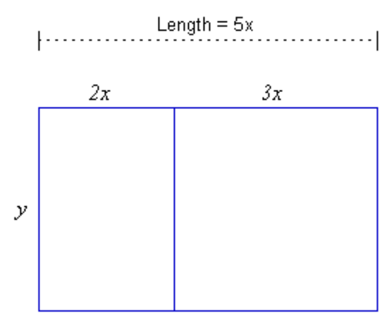
$$-4100 = -16t^2$$

$$256.25 = t^2$$

$$t = \pm 16.008$$

The pebble would take approximately 16 sec to reach the ground (the value for negative tome is discarded as not making sense).

26. Fencing goes around the perimeter.



$$P = y + (2x+3x) + y + (3x+2x)$$

$$= y + 10x$$

$$500 = y + 10x$$

Area of a rectangle = length(width)

$$A = 5xy$$

Solve perimeter equation for y $\qquad y = 500 - 10x$

Substitute into area equation

$$A = 5xy$$

$$A(x) = 5x(500-10x)$$ This function gives the area with respect to length x

$$= -50x^2 + 2500x$$

Vertex $(x, A(x))$ where x = length where the max. area occurs

$$x = \frac{-b}{2a}$$
$$= \frac{-2500}{2(-50)}$$
$$= 25$$

Length $l = 5x$ Width is y which was written $y = 500 - 10x$

$l = 5(25)$ $y = 500 - 10(25)$

$= 125$ $= 250$

The dimensions that maximize the area for the pens are 125 ft by 250 ft.

36. 2 possible solutions

Vertex $(1, -50)$ and a point $(-4, 0)$ Use vertex form

$y = a(x-h)^2 + k$ Insert the vertex

$y = a(x-1)^2 - 50$

$0 = a(-4-1)^2 - 50$

$50 = 25a$ Insert the point

$a = 2$

Equation in vertex form $y = 2(x-1)^2 - 50$

Two zeros, $x = -4$ and $x = 6$ a point $(1, -50)$ Use factored form

$y = a(x - z_1)(x - z_2)$ Insert the zeros

$y = a(x+4)(x-6)$

$-50 = a(1+4)(1-6)$

$-50 = a(5)(-5)$ Insert the point

$-50 = -25a$

$a = 2$

Equation in factored form $y = 2(x+4)(x-6)$

39. Vertex $(1,1)$ and a point $(2, 2)$ Use vertex form

$y = a(x-h)^2 + k$ Insert the vertex

$y = a(x-1)^2 + 1$

$2 = a(2-1)^2 + 1$ Insert the point

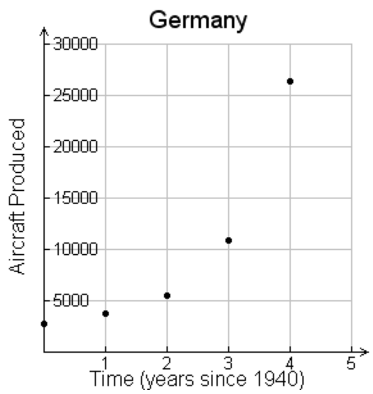
$1 = a$

Equation in vertex form $y = (x-1)^2 + 1$

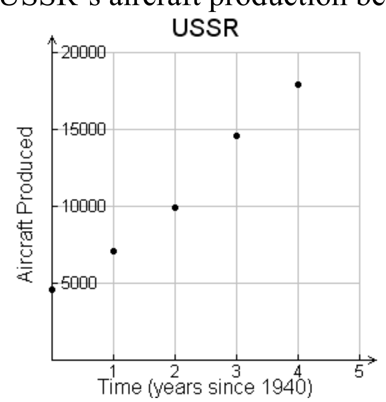
Ch. 4 Review

6. Skip

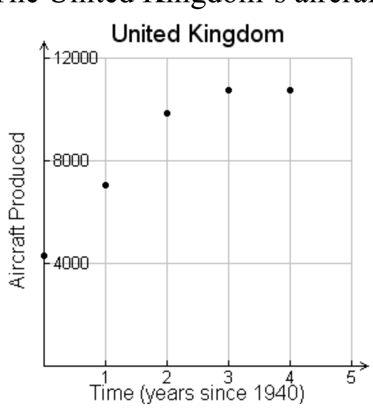
7. Germany's aircraft production between 1940 and 1944 increased at an increasing rate.



USSR's aircraft production between 1940 and 1944 increased at an nearly constant rate.



The United Kingdom's aircraft production between 1940 and 1944 increased at a decreasing rate.



8. Average Rate of Change for Germany

$$A.R.\text{ of } C. = \frac{G(4)-G(2)}{4-2}$$
$$= \frac{26326-5515}{2}$$
$$= 10405.5$$
$$\approx 10406$$

For the years 1942 to 1944, Germany's aircraft production increased approximately 10,406 aircraft per year.

11. Instantaneous Rate of Change for Germany

$$I.R.\text{ of } C. = \frac{G(4)-G(3)}{4-3}$$
$$= \frac{26326-10898}{1}$$
$$= 15428$$

For the years 1943, Germany's aircraft production increased approximately 15,428 aircraft per year.

12. Germany

For 1943-1944 $\quad \Delta G = 26326 - 10898$
$$= 15428$$

For 1945 $\quad G(4) + \Delta G = 26326 + 15428$
$$= 41754 \text{ aircraft}$$

USSR

For 1943-1944 $\quad \Delta U = 17913 - 14590$
$$= 3323$$

For 1945 $\quad U(4) + \Delta U = 17913 + 3323$
$$= 21236 \text{ aircraft}$$

United Kingdom

For 1943-1944 $\quad \Delta K = 10730 - 10727$
$$= 3$$

For 1945 $\quad K(4) + \Delta K = 10730 + 3$
$$= 10733 \text{ aircraft}$$

21. $a = 1.008$ — Rate of change in consumer expenditure = 2 (1.008) = 2.016
Based on data from 1990-2004, the rate of change in consumer expenditure is increasing at a rate of 2.016 billion dollars per year each year.

$b = 9.951$ — Based on data from 1990-2004, the rate of change in consumer expenditure for 1990 is increasing 9.951 billion dollars per year.

$c = 450.5$ — Based on data from 1990-2004, the consumer expenditure for 1990 is 450.5 billion dollars.

25. $C(t)=15.838t^2+181.401t+4218.779$ million dollars, where t = 0 represents 1990

26. $y=-10(x+1)^2-20$

Vertex $(-1,-20)$

Horizontal intercepts or zeros

$$0=-10(x+1)^2-20$$
$$20=-10(x+1)^2$$
$$-2=(x+1)^2$$
$$\pm\sqrt{-2}=x+1$$
$$x=-1\pm\sqrt{2}i$$

Since this is a complex solution, there are no real zeros, therefore this function does not cross the horizontal axis.

27. $y=4x^2+16x+30$

Vertex

$$x=\frac{-16}{2(4)}=-2 \qquad y(-2)=4(-2)^2+16(-2)+30=14 \qquad \text{Vertex is } (-2,14)$$

Horizontal intercepts or zeros

$$\begin{aligned} x&=\frac{-16\pm\sqrt{16^2-4(4)(30)}}{2(4)} \\ &=\frac{-16\pm\sqrt{-224}}{8} \\ &=\frac{-16}{8}\pm\frac{14.967i}{8} \\ &=-2\pm1.871i \end{aligned}$$

Since this is a complex solution, there are no real zeros, therefore this function does not cross the horizontal axis.

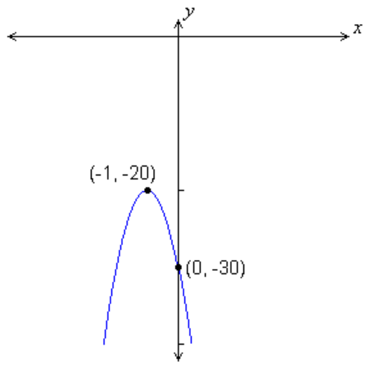
28. $y=-6(x+20)(x-8)$

Vertex will be half-way between the zeros. The zeros are $x=-20$ and $x=8$.
The distance between the zeros is 28 units. Half-way would be 14 units. By adding 14 to the zero, $x=-20$, the x-coordinate of the vertex is $x=-6$. Use this in the function to find the y-coordinate.

$$y(-6)=-6(-6+20)(-6-8)=1176 \qquad \text{Vertex is } (-6,1176)$$

Horizontal intercepts or zeros

$x=-20$ and $x=8$.

29. 

34. 2 possible solutions

Vertex $(-2,-8)$ and a point $(0,-6)$ Use vertex form

$$y = a(x-h)^2 + k$$
$$y = a(x+2)^2 - 8 \quad \text{Insert the vertex}$$
$$-6 = a(0+2)^2 - 8$$
$$2 = 4a \quad \text{Insert the point}$$
$$a = \tfrac{1}{2}$$

Equation in vertex form $y = \tfrac{1}{2}(x+2)^2 - 8$

Two zeros, $x = -6$ and $x = 2$ a point $(0,-6)$ Use factored form

$$y = a(x-z_1)(x-z_2)$$
$$y = a(x+6)(x-2) \quad \text{Insert the zeros}$$
$$-6 = a(0+6)(0-2)$$
$$-6 = a(6)(-2) \quad \text{Insert the point}$$
$$-6 = -12a$$
$$a = \tfrac{1}{2}$$

Equation in factored form $y = \tfrac{1}{2}(x+6)(x-2)$

35. Vertex $(0,8)$ and a point $(1,7)$ Use vertex form

$$y = a(x-h)^2 + k$$
$$y = a(x-0)^2 + 8 \quad \text{Insert the vertex}$$
$$= ax^2 + 8$$
$$7 = a(1)^2 + 8 \quad \text{Insert the point}$$
$$-1 = a$$

Equation in vertex form $y = -x^2 + 8$